Maneuvers uncertainties during launch, station-keeping and reentry

Introduction

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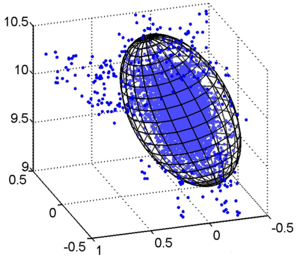
29-09-2022

Satellite uncertainties

- State vector partially unknown
- Represented by a covariance matrix (σ, ρ)
- Probability of presence in an ellipsoid
  - → 1-σ ellipsoid ⇔ 20% chance of presence
  - → 3-σ ellipsoid ⇔ 97% chance of presence



State vector uncertainties can come from different sources, such as:

- Orbit restitution or propagation errors
- Imperfect maneuvers performed during operations

Normal or Gaussian distribution law $\mathcal{N}(\mu, \sigma)$

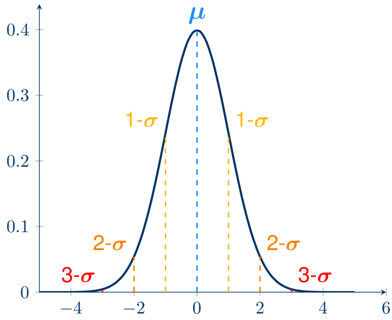


Figure: Gaussian distribution $\mathcal{N}(0, 1)$ in 1D

Probability of presence in N-σ enveloppe

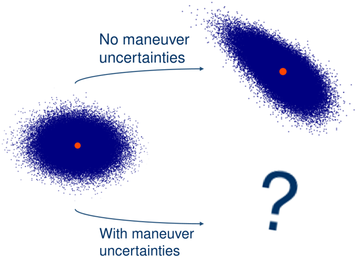
$$P = \int_0^{N^2} \frac{x^{\frac{D}{2}-1} e^{-\frac{x}{2}}}{2^{\frac{D}{2}} \Gamma\left(\frac{D}{2}\right)} dx$$

with D the dimension of the enveloppe

		1-σ	2-σ	3-σ
dimension	1	68.2%	95.5%	99.7%
	2	39.3%	86.5%	98.9%
	3	19.9%	73.9%	97.1%

Two problematics to explore

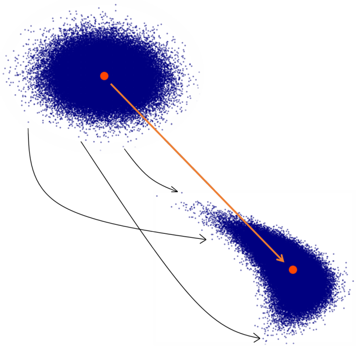
- Modeling maneuver uncertainties as a matrix
- Impact of maneuver uncertainties on final result



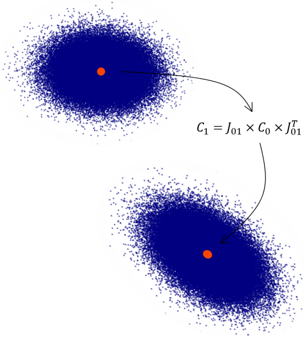
GUEPARD
Software specialized in uncertainties propagation

Two propagation methods

Monte Carlo sampling



STM (jacobian) transformation



$$C_1 = J_{01} \times C_0 \times J_{01}^T$$

Which propagation and when?

- **Monte Carlo**
  Shows an accurate distribution of the sampled uncertainties, used to estimate the timespan where uncertainties are Gaussian
- **Jacobian**
  On appropriate timespan, provides adequate matrix representation of uncertainties for collision risk computation

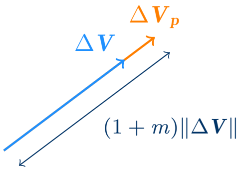
Maneuver uncertainties

Monte Carlo	Jacobian
Random error on each sample	Translate uncertainties as a matrix

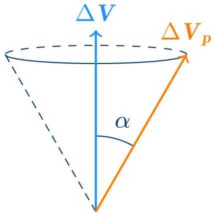
Maneuvers uncertainties

Considering a maneuver of thrust vector $\Delta \boldsymbol{V}$:

- Magnitude error $m \in \mathcal{N}\left(0, \sigma_{\text{mag}}\right)$
  - σ_{mag} given as percentage



- Direction error $\alpha \in \mathcal{N}\left(0, \sigma_{\text{dir}}\right)$
  - σ_{dir} given as an angle



Representation of maneuver uncertainties

Formula from Gates technical report[1] valid only for small values of σ_{dir}

$$\boldsymbol{M} = \left(\sigma_{\mathrm{mag}}^2 - \frac{1}{2}\sigma_{\mathrm{dir}}^2\right)\Delta\boldsymbol{V}\Delta\boldsymbol{V}^T + \frac{1}{2}\sigma_{\mathrm{dir}}^2\|\Delta\boldsymbol{V}\|^2\boldsymbol{I}$$

Formula generalized for all values of σ_{dir}

$$\boldsymbol{M} = \|\Delta\boldsymbol{V}\|^2 \begin{bmatrix} \gamma\left(1+\mathrm{e}^{-2\sigma_{\mathrm{dir}}^2}\right) - \mathrm{e}^{-\sigma_{\mathrm{dir}}^2} & 0 & 0 \\ - & \frac{1}{2}\gamma\left(1-\mathrm{e}^{-2\sigma_{\mathrm{dir}}^2}\right) & 0 \\ - & - & \frac{1}{2}\gamma\left(1-\mathrm{e}^{-2\sigma_{\mathrm{dir}}^2}\right) \end{bmatrix}$$

with

$$\gamma = \frac{1}{2}\left(1+\sigma_{\mathrm{mag}}^2\right)$$

[1] **Gates, A simplified model of midcourse maneuver execution errors, 1963**.

Problematics check

- Modeling maneuver uncertainties as a matrix ✓
- Impact of maneuver uncertainties on final result ✗

Impact on the uncertainties propagation
$\Rightarrow$ very specific to every case

- Size of initial uncertainties
- Type of maneuvers
- Number of maneuvers, magnitude, uncertainty values...

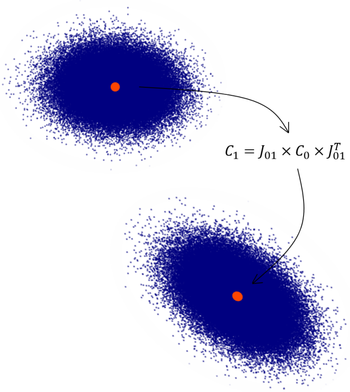
One common impact: **shift** on the nominal trajectory

Nominal trajectory = trajectory of **unperturbed** object

STM (Jacobian) propagation

Assumption that uncertainties are centered around the nominal trajectory

$$\boldsymbol{P}_{\text{nom}} = \frac{1}{N}\sum_{0}^{N}\boldsymbol{P}_{\text{pert}}$$



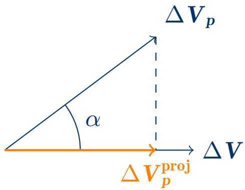
Considering a maneuver thrust $\boldsymbol{\Delta V}$

- If only **magnitude** perturbation is involved
  Average perturbed vector is $\boldsymbol{\Delta V}$

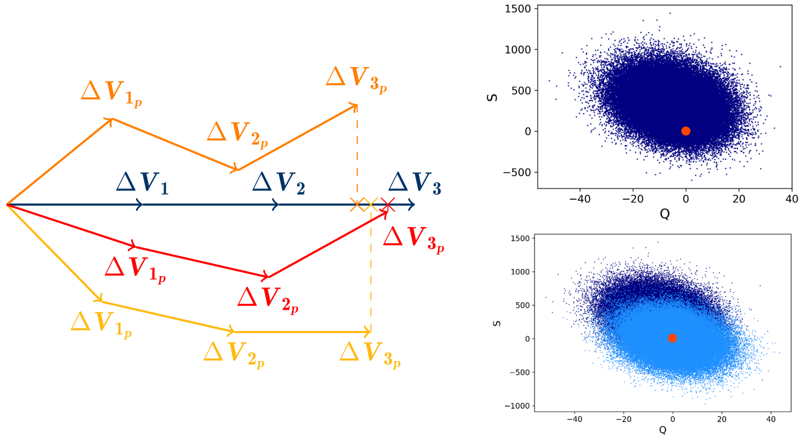
- If only **direction** perturbation is involved
  Average perturbed vector is < $\boldsymbol{\Delta V}$

$$\forall \alpha \in \mathcal{N}\left(0, \sigma_{\mathrm{dir}}\right)$$

$$\|\boldsymbol{\Delta V}_{\boldsymbol{p}}^{\mathbf{proj}}\| \leq \|\boldsymbol{\Delta V}\|$$

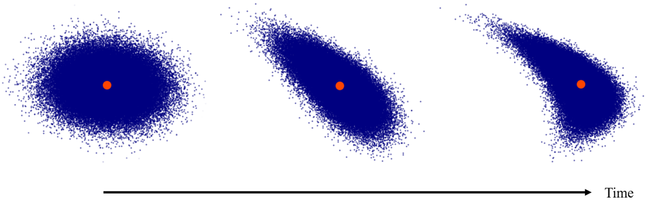


Accumulation of maneuvers with σ_{dir}



Other impact:

Loss of Gaussianity in Cartesian coordinates

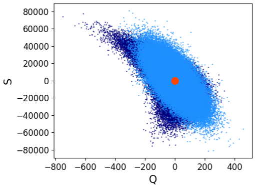


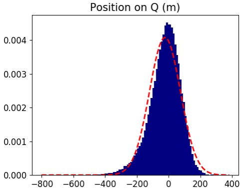
⇒ Matrix not suitable to represent the uncertainties

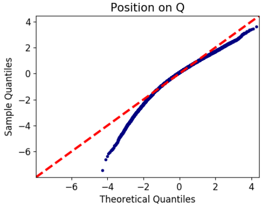
GAUSSIANITY TEST

Gaussianity verification methods

- Normality tests
- Visual comparison of **Monte Carlo** and **STM** distributions
- Histograms et Quantile-Quantile diagrams



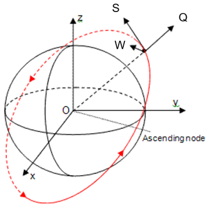




QtW coordinates

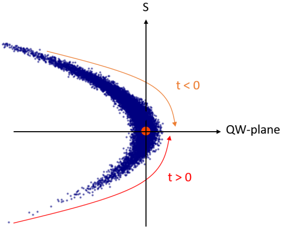
QSW = local orbital frame

Q $\rightarrow$ radial
S $\rightarrow$ pseudo-tangential
W $\rightarrow$ out of plane



Time component?

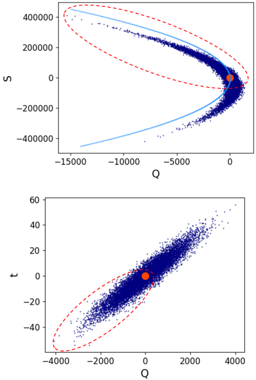
Propagation from perturbed sample to nominal QW-plane



QTW COORDINATES

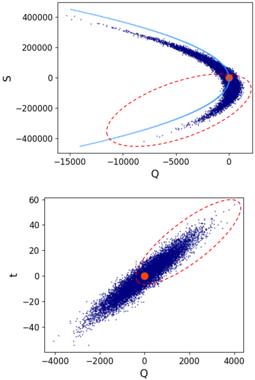
Samples *in front*:

- under nominal trajectory
- higher speed
- in advance wrt the nominal sample
  $\Rightarrow t < 0$



Samples *behind*:

- above nominal trajectory
- lower speed
- late compared to nominal sample
  $\Rightarrow t > 0$



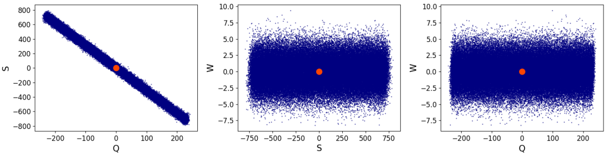
Limitations of QtW for risk computation

- Secondary object covariance can't be taken into account

  $\Rightarrow$ $C_{\text{sec}} << C_{\text{prim}}$

- Transformation of *curved* uncertainties

  $\Rightarrow$ Uncertainties shaped along the orbit



Loss of Gaussianity unrelated to orbit curve $\rightarrow$ QtW useless

Problematics check

- Modeling maneuver uncertainties as a matrix ✓
- Impact of maneuver uncertainties on final result ✓
  Stay tuned for more information...

Thank you for your attention

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