

Simulating displacement damages in electronic devices: from primary knock-on atom to electronic noise

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⁴CNR-IOM, *Democritos and Sissa, Trieste, Italy*

⁵CEA, DAM, DIF, *Arpajon, France*

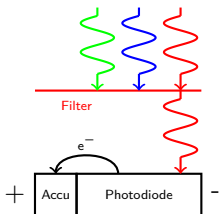
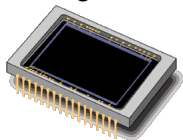
⁶ZTF-IRB, *Zagreb, Croatia*

⁷CNES, *Toulouse, France*

November 29th 2023

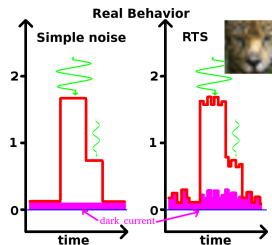
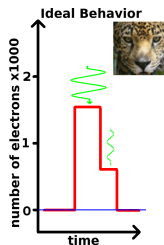
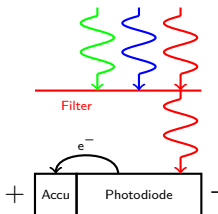
Context: leakage current, CMOS example

CMOS
Image Sensor



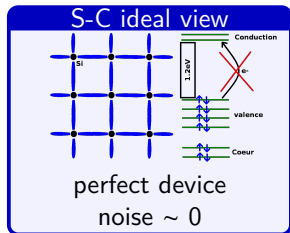
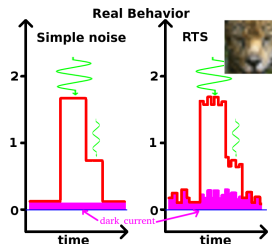
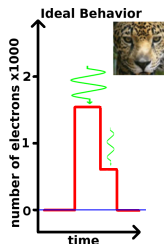
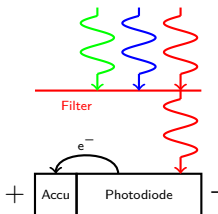
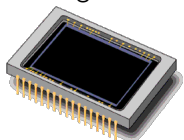
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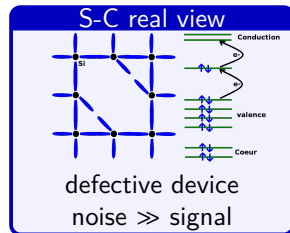
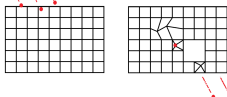


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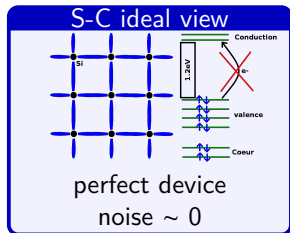
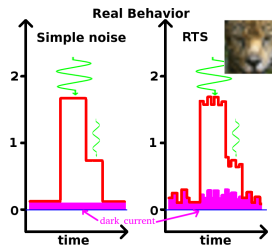
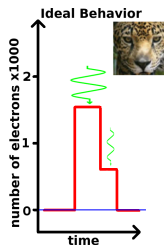
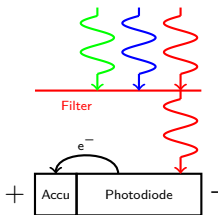
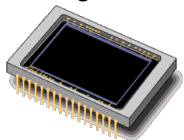


irradiation
implantation
deposition

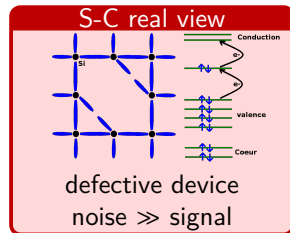
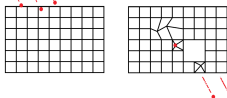


Context: leakage current, CMOS example

CMOS
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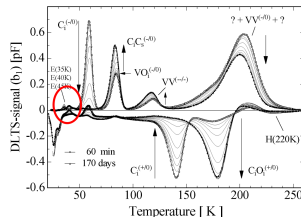
irradiation
implantation
deposition



Unpredictable: technological problem, challenging modelization

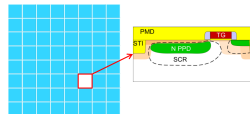
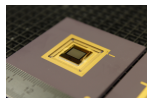
Why studying defects with image sensors?

- Photoluminescence,
- Electron Paramagnetic Resonance
- Deep Level Transient Spectroscopy
- Raman and Infra-Red
 - stable defects
 - many IDENTICAL defects: $10^9/\text{cm}^3$



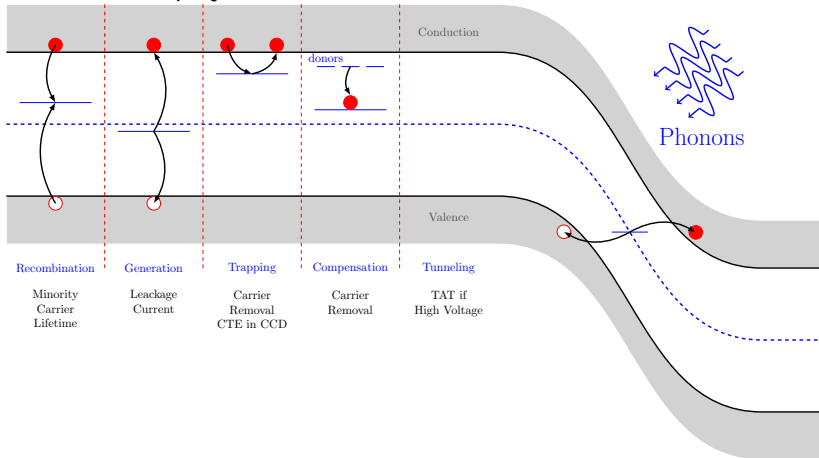
M. Moll, PhD thesis, 1999

- Monitoring pixel array
 - metastable defects
 - 1 single defect
 - Statistics



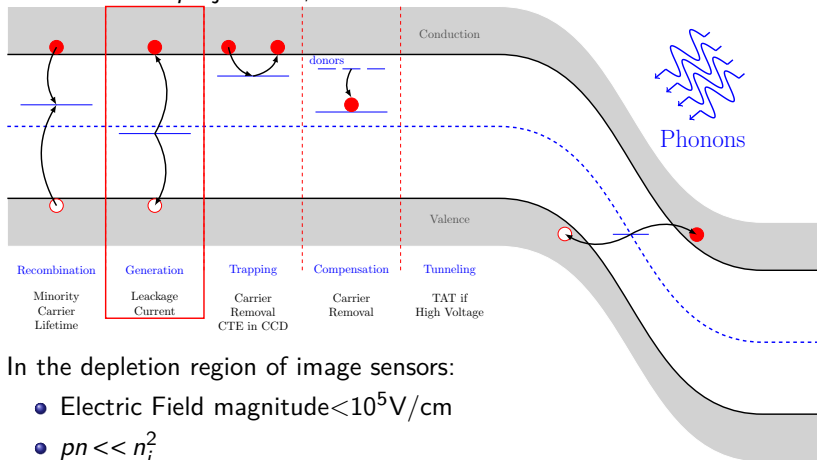
Focus of the talk

Effect of Displacement Damage (not Total Ionizing Dose) Reverse biased *pn* junction, bulk Silicon



Focus of the talk

Effect of Displacement Damage (not Total Ionizing Dose) Reverse biased *pn* junction, bulk Silicon



In the depletion region of image sensors:

- Electric Field magnitude $< 10^5 \text{ V/cm}$
- $pn \ll n_i^2$
- 1 single defect generation dominates the signal

Overview

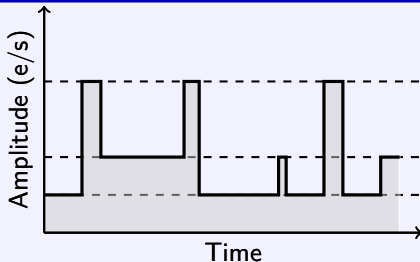
- 1 Observations
 - Characteristics
 - Independant parameters
- 2 Simulations
 - GEANT4
 - DM
 - kART
 - DFT
 - Electronic cross sections
- 3 conclusion
- 4 back up

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Random Telegraph Signal characteristics

Measured RTS

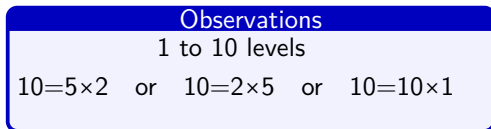
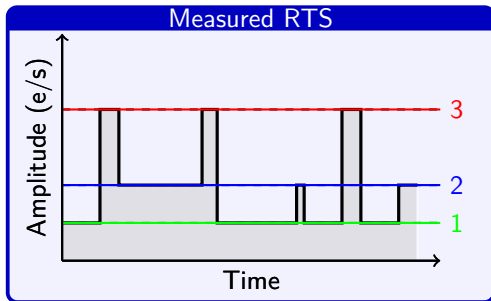


Characteristics:

- Number of levels
- Life times
- Activation Energies
- Amplitudes

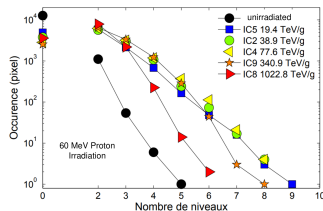
Observations

Random Telegraph Signal characteristics



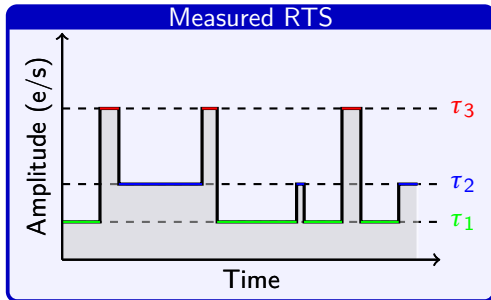
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C. Virmondois, PhD thesis, 2012

Random Telegraph Signal characteristics



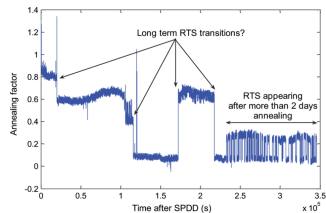
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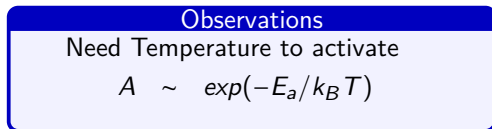
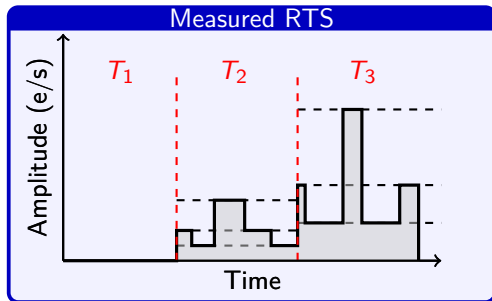
From ms to months

$$1/\tau \sim \exp(-E/k_B T)$$



M. Raine *et al.*, IEEE TNS, 2014

Random Telegraph Signal characteristics



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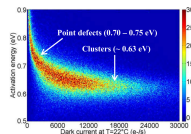
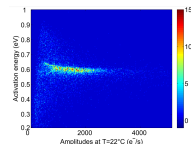


Fig. 3. Dark current activation energy in CIS 2 (10^{12} cm $^{-2}$ 22 MeV neutrons). The color scale corresponds to the pixel occurrence.

Cst DC

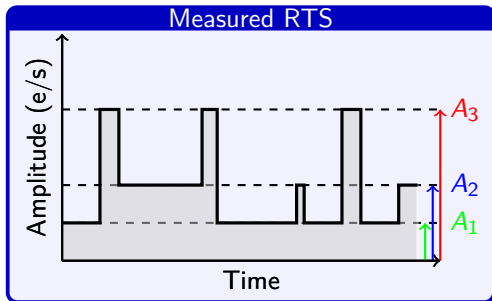
J.M. Belloir, PhD thesis, 2017



DC RTS

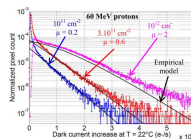
C. Durnez, PhD thesis, 2017

Random Telegraph Signal characteristics



Characteristics:

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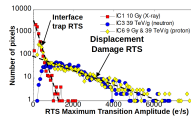
Cst DC

Observations

$$A_i - A_j > 10^3 \text{ e/s}$$

Predictable exp. shape: $f(\text{DD}, V_{\text{dep}})$

J.M. Belloir, PhD thesis, 2017



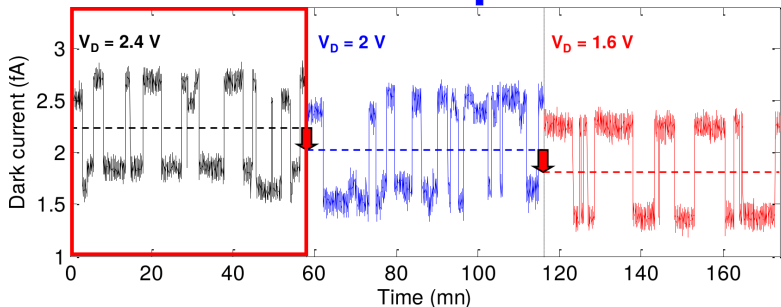
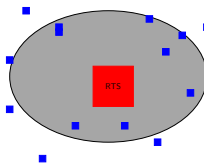
DC RTS

C. Virmondois, PhD thesis, 2012

Effect of electric field?

Small Voltage + low doping (no tunneling)

- Mean DC decreases
- No change in RTS amplitudes



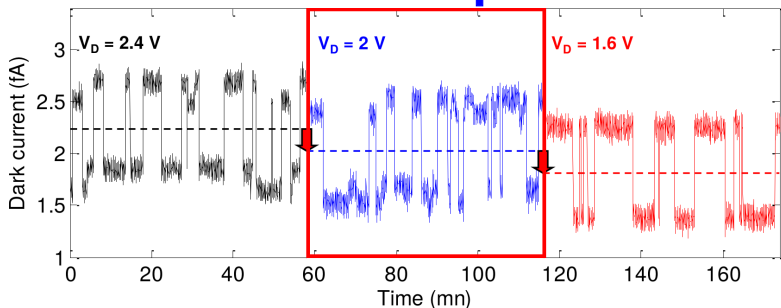
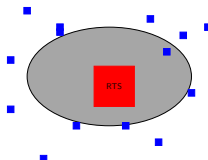
see C. Durnez, PhD thesis, 2017 for larger voltages

Electric field not needed to explain large RTS relative amplitudes.

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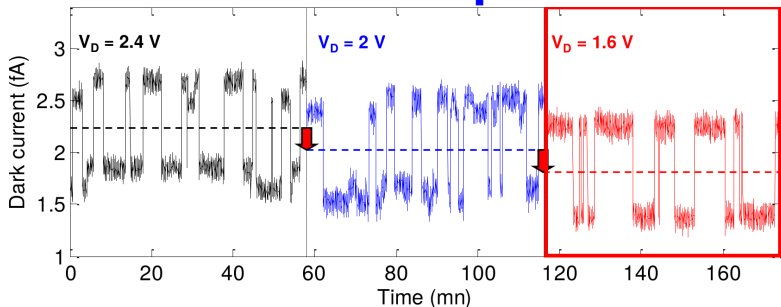
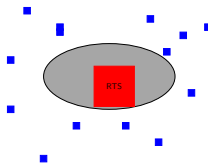
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Effect of electric field?

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see C. Durnez, PhD thesis, 2017 for larger voltages

Electric field not needed to explain large RTS relative amplitudes.

Technology? Material?

All the components:

PV cells



CMOS



Fiber



DRAM



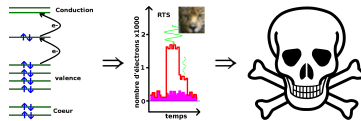
IR sensors



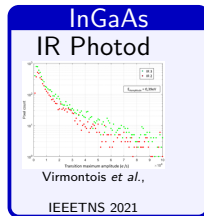
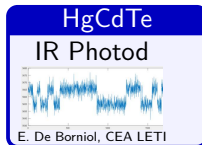
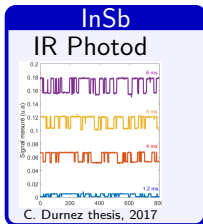
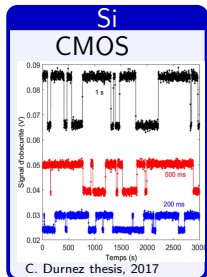
SPAD



Same phenomenon:



↘ efficacité, ↘ life time



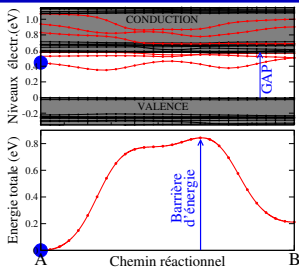
Constant and RTS leakage currents are everywhere

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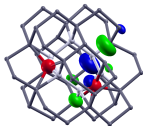
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State of the art

ab initio capacities

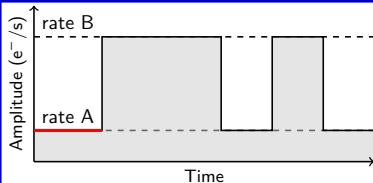


metastable



A

Observation in devices



CMOS
example
Pixel flashes

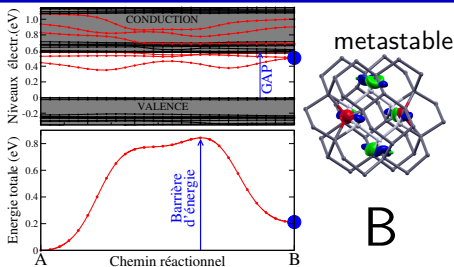


structure variation \Rightarrow noise variation=RTS

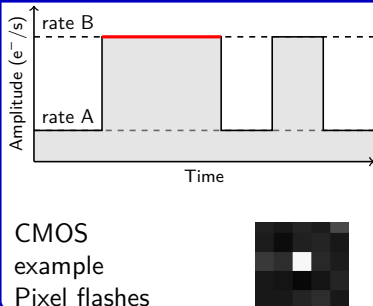
Challenge: Quantify the number of generated electrons

State of the art

ab initio capacities



Observation in devices

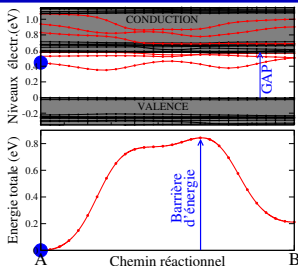


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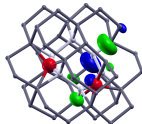
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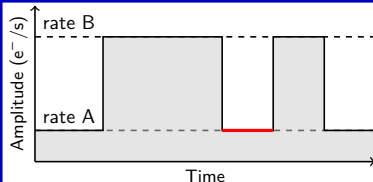


metastable



A

Observation in devices



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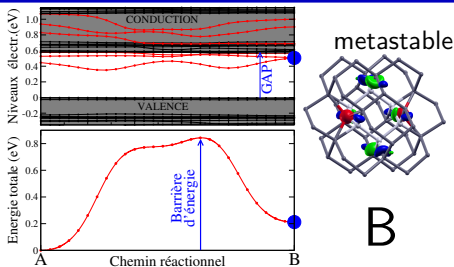


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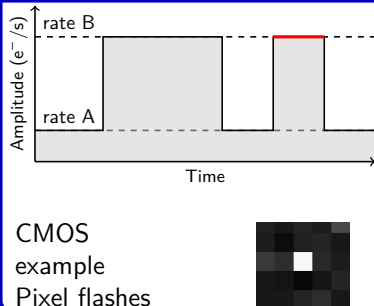
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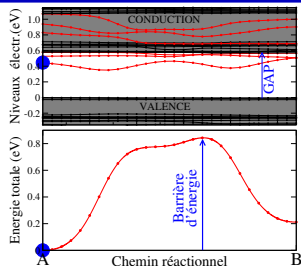


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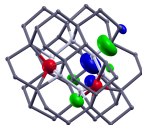
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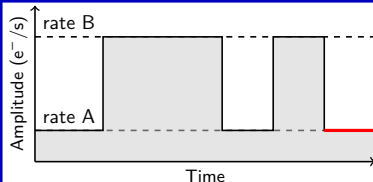


metastable



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Observation in devices



CMOS
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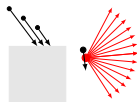
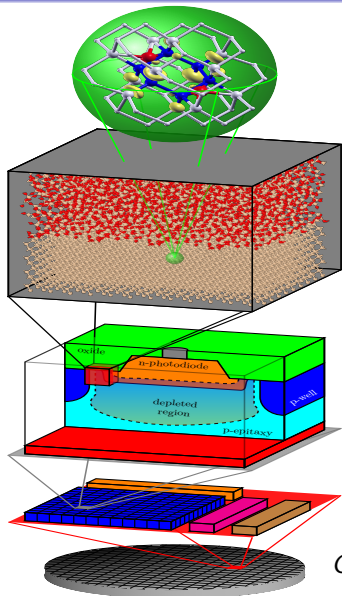


structure variation \Rightarrow noise variation=RTS

Challenge: Quantify the number of generated electrons

Strategy to simulate the impact of irradiation/implantation

Problem : Apply Shockley Read Hall



Neutrons,
Dopants
...
KMC

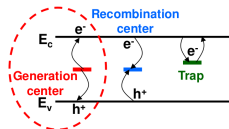
Rapidity Vs Precision

- Incomplete mechanisms
- Bad time scale
- Not transportable

No atomic structure

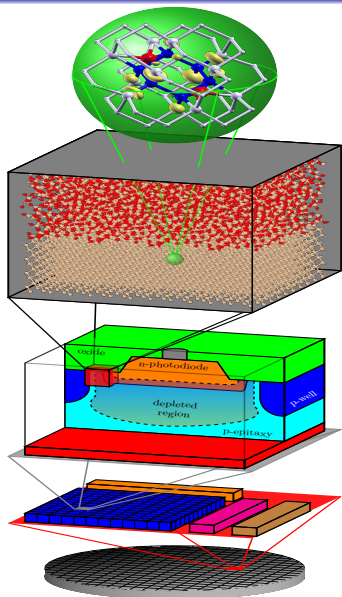


E-h:
generation
rate



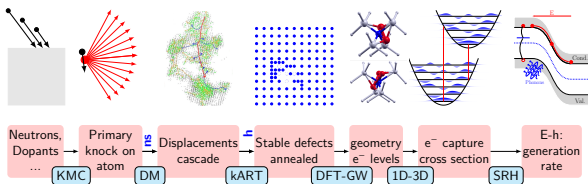
$$G_{SRH} = \frac{v_{th} \sigma_n \sigma_p (n_i^2 - np)}{\sigma_n [n + n_i \exp(\frac{E_t - E_F}{k_B T})] + \sigma_p [p + n_i \exp(\frac{E_t - E_F}{k_B T})]}$$

Strategy to simulate the impact of irradiation/implantation

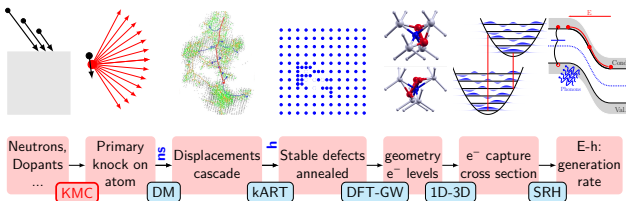


Problem : Apply Shockley Read Hall

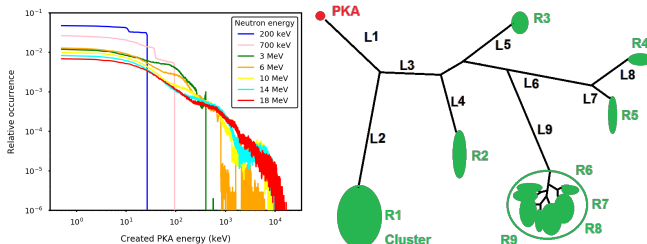
Solution: Multi-scale model



Step 1: get PKA energies



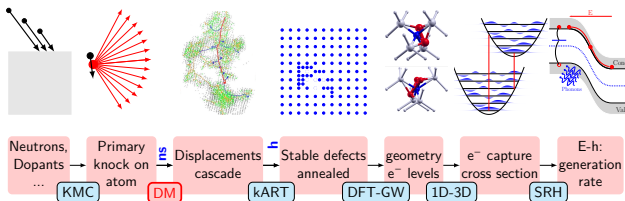
GEANT4: binary collision approximation in $\text{Si}_{0.8}\text{Ge}_{0.2}$



Raine *et al.*, IETNS 2017

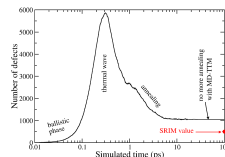
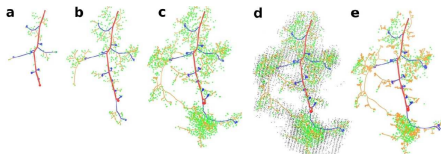
mean free path = cluster size \rightarrow overlap

Step 2: Collision Cascades - method



- Solve equation of motion
- 1 interatomic potential
- 1 million crystalline Si atoms
- 10 keV PKA

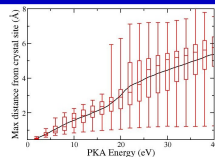
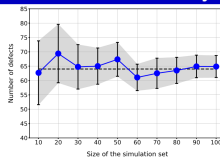
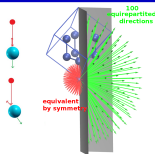
A. Jay, IETNS 2017



SRIM?

Step 2: Collision Cascades - improved

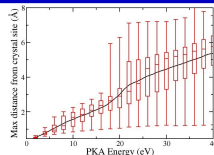
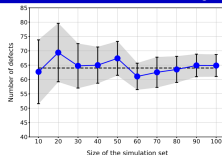
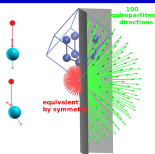
Stochasticity



T. Jarrin *et al.*, NIMB 2021

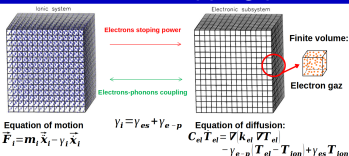
Step 2: Collision Cascades - improved

Stochasticity



T. Jarrin *et al.*, NIMB 2021

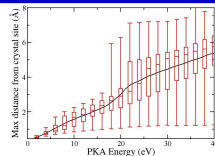
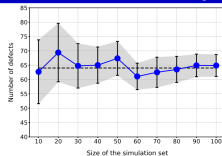
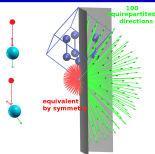
Electronic Stopping power



T. Jarrin *et al.*, NIMB 2020

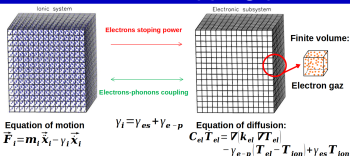
Step 2: Collision Cascades - improved

Stochasticity



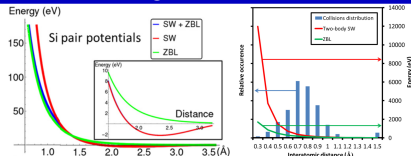
T. Jarrin *et al.*, NIMB 2021

Electronic Stopping power



T. Jarrin *et al.*, NIMB 2020

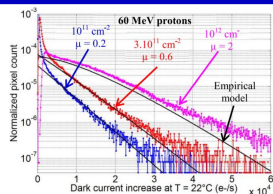
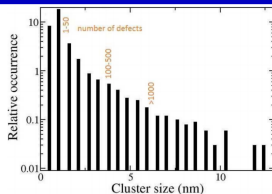
Short Range interaction Potential




T. Jarrin *et al.*, IETNS 2020

Step 2: Collision Cascade - results

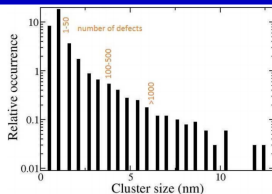
Exponential Shape



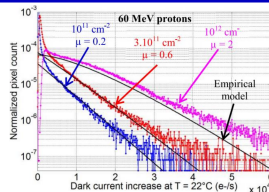
A. Jayet *al.* IEETNS 2017  correlation \neq causality \rightarrow need more

Step 2: Collision Cascade - results

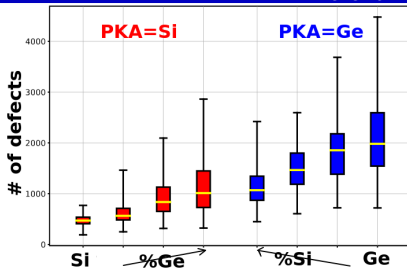
Exponential Shape



A. Jayet *al.* IETNS 2017  correlation \neq causality \rightarrow need more



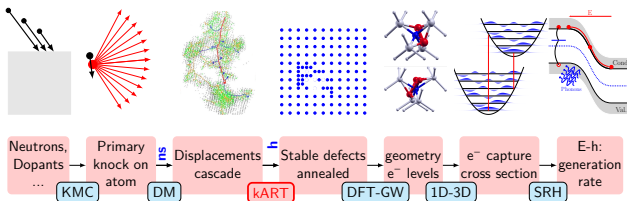
Different materials



- Ge PKA damage more the material than Si PKA
- The greater the % of Ge, the more defects are produced

T. Jarrin, PhD Thesis, 2021

Step 3: Long time annealing of the defects - method



Potential Energy Surface

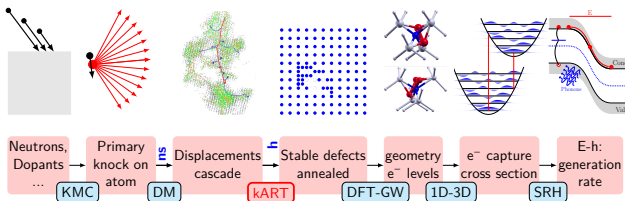
Find Transition State

- Stable structures = minima
- Transition states = saddle point
- Activation Relaxation Technique
- Init+Saddle+Final= event

A. Jay *et al.*, JCTC 2020

A. Jay *et al.*, CMS 2022

Step 3: Long time annealing of the defects - method



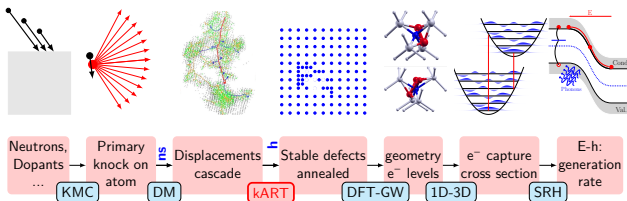
Potential Energy Surface

Stabilize the system

- Start from DM last structure
- Find all atomic events
- Choose 1 event and apply it
- Do it again

L.K. Béland *et al.*, PRE 2011

Step 3: Long time annealing of the defects - method



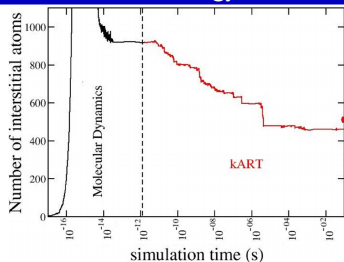
Structure evolution

- Change atomic positions
- Flickers appears = metastable states

A. Jay *et al.*, IETNS 2016

Step 3: Long time annealing of the defects - results

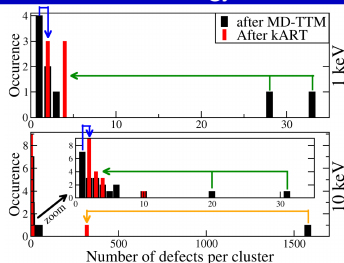
Potential Energy Surface



A. Jayet *al.*, IEETNS 2016

- Reach correct # of defects
- Reach experimental time scales

Potential Energy Surface



A. Jayet *al.*, IEETNS 2016

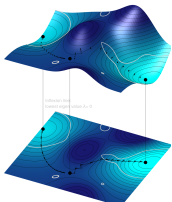
- Reduction of the clusters size
- Diffusion agglomeration of point defects

Step 3: Long time annealing of the defects - results

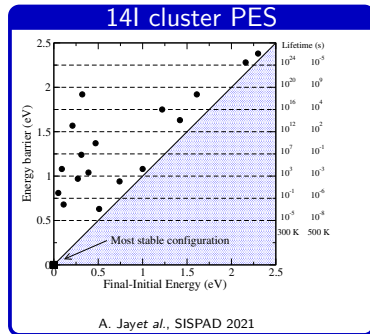
Number of levels problem!

- point defects are simple:
small number of topologies
- Clusters are complicated:
many topologies

1 RTS level = 1 stable configuration

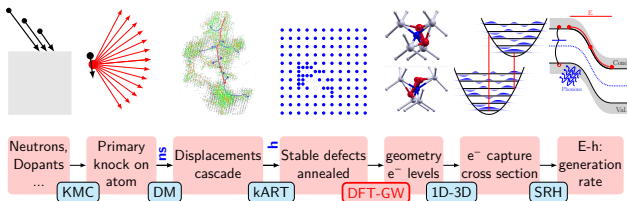


PES Exploration
with ARTn-DFT



A cluster exists in many different configurations!

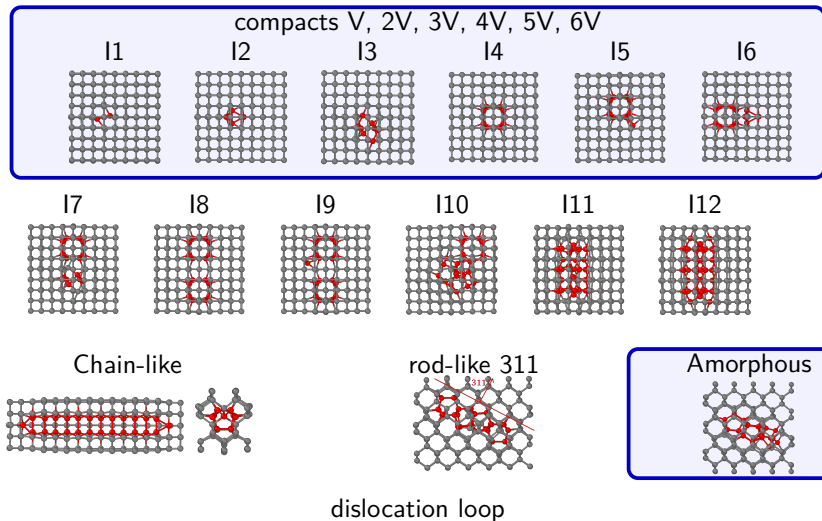
Step 4: *Ab initio* - method



solve Hamiltonian $H|\Psi_i\rangle = E|\Psi_i\rangle$

- Get total E and Forces → minimization
- Get electronic states (corrected by GW)

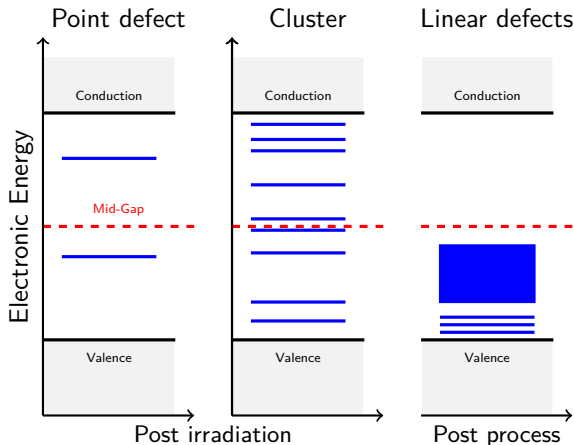
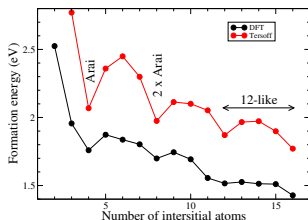
Step 4: *Ab initio* - Si defects bestiary



⚠ POST-IRRADIATION ≠ POST-IMPLANTATION+ANNEALING

Step 4: *Ab initio* - results on annealed clusters

Lower E_f = more stable



Step 4: *Ab initio* - results on simple DC

Case: simple DC

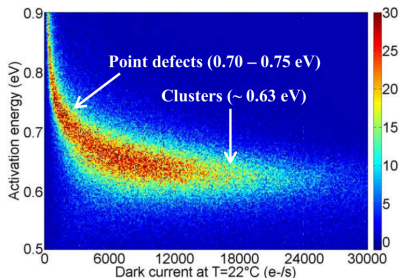


Fig. 3. Dark current activation energy in CIS 2 (10^{12} cm^{-2} 22 MeV neutrons). The color scale corresponds to the pixel occurrence.

Proba ~ 0

Proba = ?

$$E_{act} > \frac{E_{GAP}}{2}$$

WORST= midgap

Observed crossing through 1 level

Step 4: *Ab initio* - results on simple DC

Case: simple DC

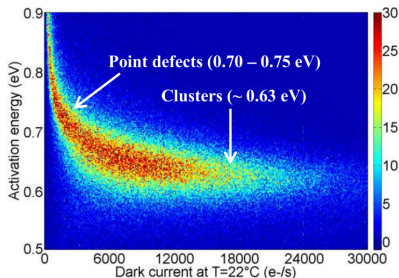
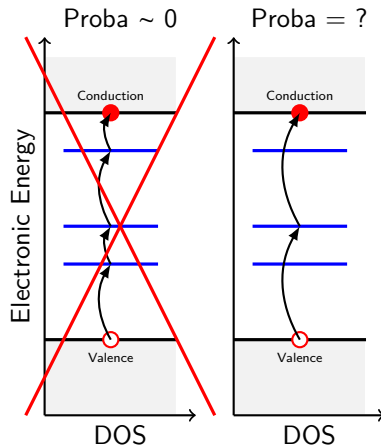


Fig. 3. Dark current activation energy in CIS 2 (10^{12} cm^{-2} 22 MeV neutrons). The color scale corresponds to the pixel occurrence.

$$E_{act} > \frac{E_{GAP}}{2}$$

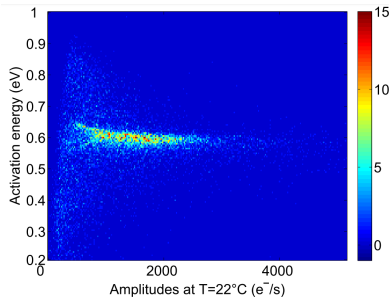
WORST= midgap



Observed crossing through 1 level

Step 4: *Ab initio* - results on DC-RTS

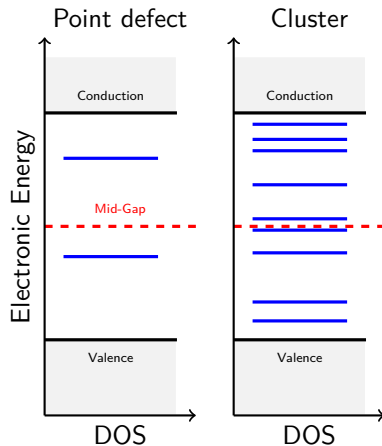
Case: DC-RTS



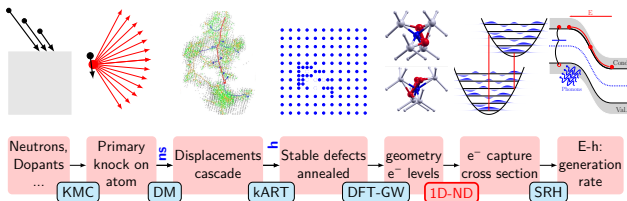
$$E_{act} \sim \frac{E_{GAP}}{2}$$

Many levels → 1 is midgap

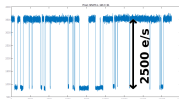
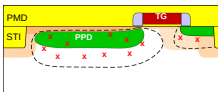
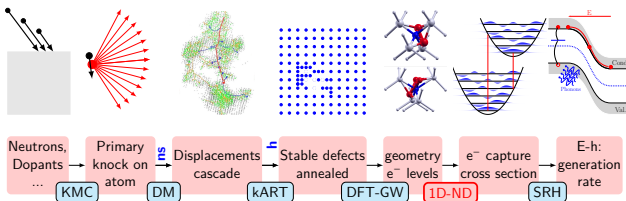
Clusters have many electronic levels in the band gap



Step 5: Electronic capture rate - Amplitude problem



Step 5: Electronic capture rate - Amplitude problem



SRH generation rate:

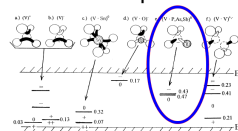
$$A \approx \frac{\sigma v_{th} n_i}{2 \cosh\left(\frac{E_t - E_{mg}}{k_B T}\right)}$$

$\sigma = e^-$ cross section

$$v_{th} = 2.7 \times 10^7 \text{ cm/s}$$

$$n_i = 1.45 \times 10^{10} / \text{cm}^3$$

Worth known point defect

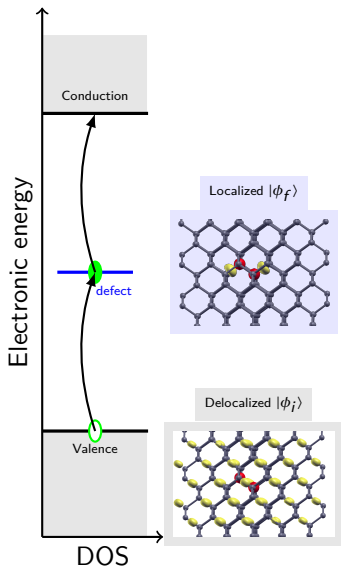


PV:Phosphorus-Vacancy in Silicon

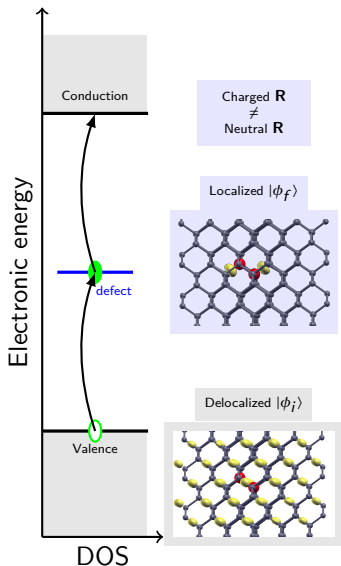
$$\left. \begin{array}{l} \sigma = 10^{-15} \text{ cm}^2 \\ E_t - E_{mg} = 0.07 \text{ eV} \end{array} \right\} 20 \text{ e/s (300 K)}$$

Need larger σ

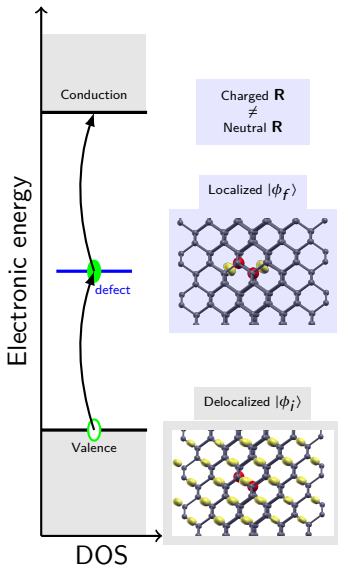
Step 5: multiphonon excitation - method



Step 5: multiphonon excitation - method



Step 5: multiphonon excitation - method



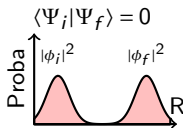
Charged R
≠
Neutral R

Localized $|\phi_f\rangle$

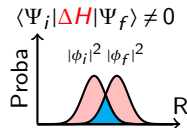
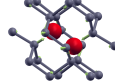
Delocalized $|\phi_i\rangle$

Fermi's Golden rule = overlap

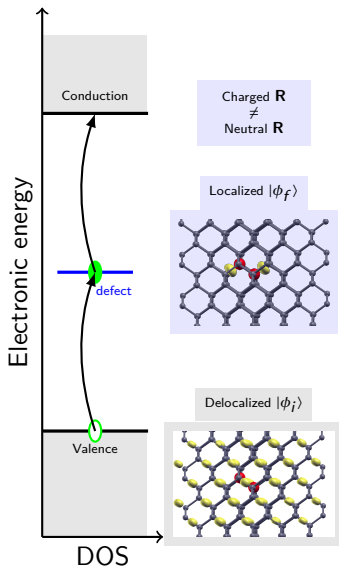
$$\Gamma_{i \rightarrow f} \sim |\langle \Psi_i | \Delta H | \Psi_f \rangle|^2$$



Vibrations
Phonons



Step 5: multiphonon excitation - method



Fermi's Golden rule = overlap

$$\Gamma_{i \rightarrow f} \sim |\langle \Psi_i | \Delta H | \Psi_f \rangle|^2$$



Born-Oppenheimer

$$\Psi_n(\mathbf{r}, \mathbf{R}) = \phi_n(\mathbf{r}, \mathbf{R}) \chi_n(\mathbf{R})$$

Taylor 1st order

$$\Gamma_{i \rightarrow f} \sim \left| \sum_k N_{modes} \underbrace{\langle \phi_i | \frac{\partial H}{\partial \mathbf{Q}_k} | \phi_f \rangle}_{\text{electrons}} \underbrace{\langle \chi_{im} | \Delta \mathbf{Q}_k | \chi_{fn} \rangle}_{\text{ions}} \right|^2 \Delta E$$

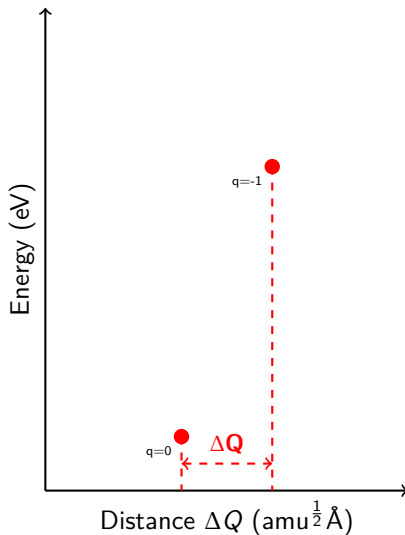
need ~ 10 phonons \rightarrow multiphonon

Step 5: Electronic part - method : 1D model

Approximation:

1 fictive phonon is responsible of the transition

- Relax \mathbf{R}_i
at $q=0$ and $q=1$
- E_{tot}
for $\mathbf{R}_{0 \rightarrow 1}^{q=0}$ and $\mathbf{R}_{0 \rightarrow 1}^{q=1}$
- Fit the curvature:
 $\omega = \frac{\partial^2 E}{\partial Q^2}$
- Deduce number of needed phonons $n\hbar\omega$
- $\langle \phi_i | \frac{\partial H}{\partial Q} | \phi_f \rangle$
(back up slides)

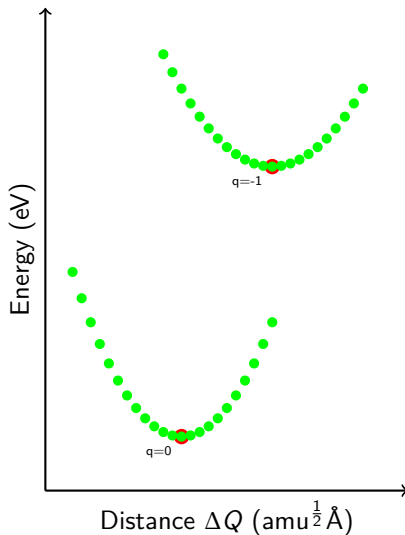


Step 5: Electronic part - method : 1D model

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(back up slides)

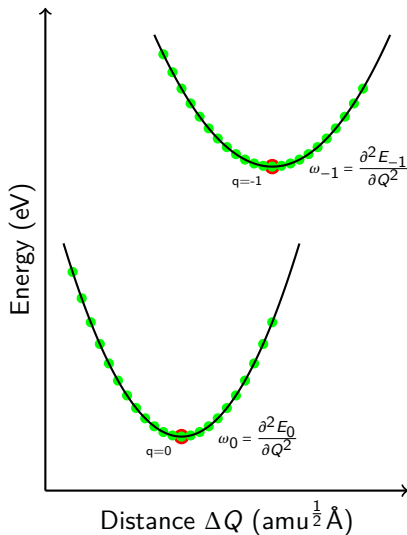


Step 5: Electronic part - method : 1D model

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- E_{tot}
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(back up slides)

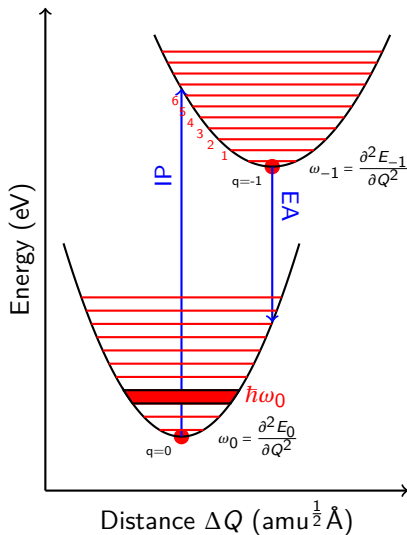


Step 5: Electronic part - method : 1D model

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(back up slides)

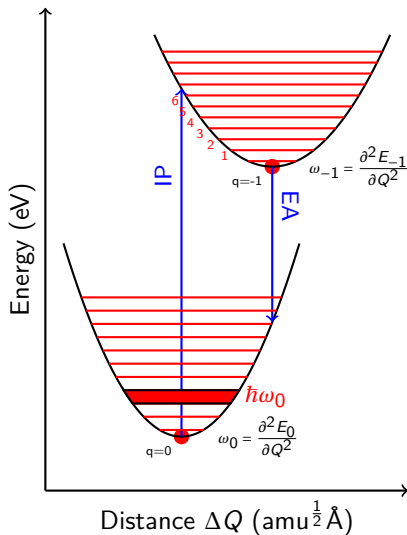


Step 5: Electronic part - method : 1D model

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- Deduce number of needed phonons $n\hbar\omega$
- $\langle \phi_i | \frac{\partial H}{\partial Q} | \phi_f \rangle$
(back up slides)



Step 5: Ionic part - method 1D vs ND

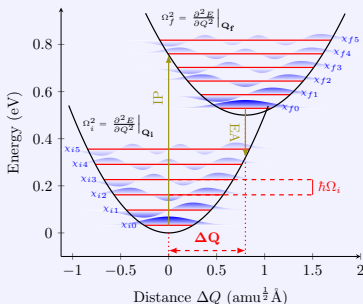
$$H^{ion}|\chi_n(\mathbf{R})\rangle = E_n|\chi_n(\mathbf{R})\rangle$$

1 dimension

Analytic solutions

Hermite polynomials order n

$$n = \frac{1}{\exp(\hbar\omega/k_B T) - 1} \text{ (BE)}$$



N dimensions

Sol: 2nd quantization

Huang, Prog. Phys., **1**, 31 (1981)

Borrelli, J. Phys. Chem. A **116**, 9934 (2012)

No direct solution

Xiao, Sci. China Phys. Mech. Astron. **63**, 277312 (2020)

Work in progress (backup)

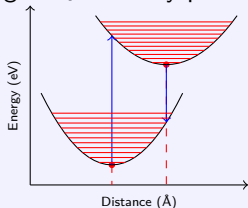
A. Jay *et al.*, SISPAD 2023

Step 5: Electronic cross section - results

$$\sigma_{cluster} \sim 1000 \sigma_{single\ defect}$$

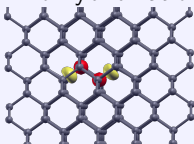
Point defects

Large $\Delta Q \Rightarrow$ many phonons



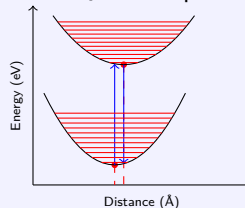
Reason: Localized $|\phi_i\rangle$

No hybridization



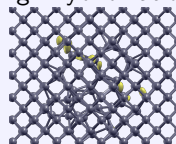
Clusters of defects

Small $\Delta Q \Rightarrow < 1$ phonon

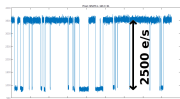
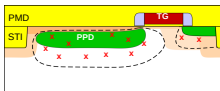


Reason: Extended $|\phi_i\rangle$

high hybridization



Step 5: Electronic cross section - results

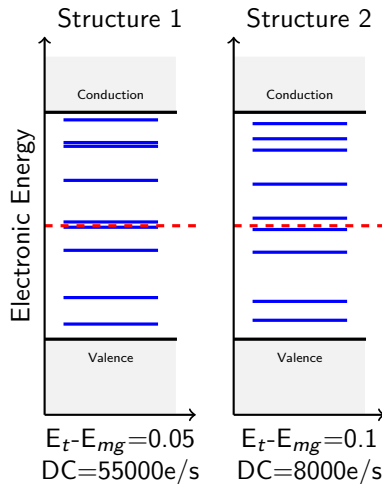


SRH generation rate:

$$DC_{SRH} \approx \frac{\sigma v_{th} n_i}{2 \cosh\left(\frac{E_t - E_{mg}}{k_B T}\right)}$$

Large $\sigma = 10^{-12} \text{ cm}^2$

small structure change \Rightarrow
small elec. level change \Rightarrow
Large DC variation



Solve Amplitude problem

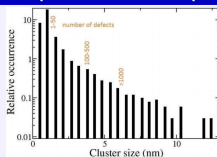
Table of Contents

- 1 Observations
 - Characteristics
 - Independant parameters
- 2 Simulations
 - GEANT4
 - DM
 - kART
 - DFT
 - Electronic cross sections
- 3 conclusion
- 4 back up

Conclusion - Question answered

WHY clusters of defects correspond to observed DC and DC-RTS ?

Exponential Shape

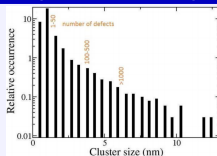


Same amplitude distribution

Conclusion - Question answered

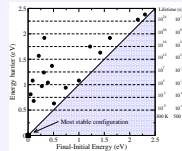
WHY clusters of defects correspond to observed DC and DC-RTS ?

Exponential Shape



Same amplitude distribution

Highly Metastable

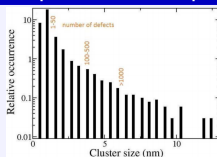


Multilevel RTS, all τ

Conclusion - Question answered

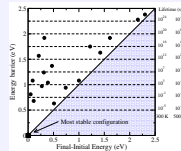
WHY clusters of defects correspond to observed DC and DC-RTS ?

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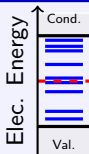
Same amplitude distribution

Highly Metastable



Multilevel RTS, all τ

Many electronic states

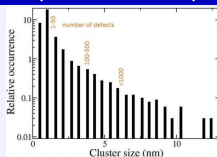


Midgap activation energies

Conclusion - Question answered

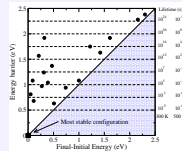
WHY clusters of defects correspond to observed DC and DC-RTS ?

Exponential Shape



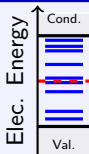
Same amplitude distribution

Highly Metastable



Multilevel RTS, all τ

Many electronic states



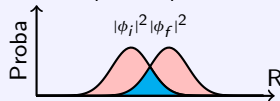
Midgap activation energies

Large cross sections

small ΔR

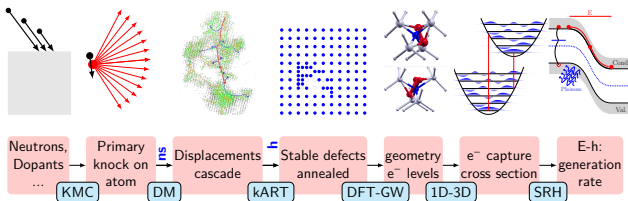
$$\langle \Psi_i | \Delta H | \Psi_f \rangle \neq 0$$

$$|\phi_i|^2 |\phi_f|^2$$



Large variation of amplitude

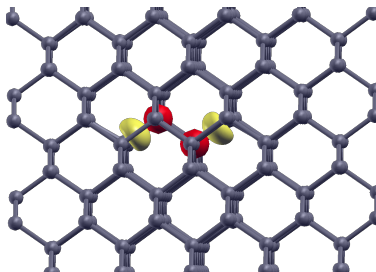
Conclusion - ongoing works



- Si & Ge Vs large (GaN, SiC) and small (InGaAs, HgCdTe) gap sc
- Ponctual defects → RTS?
- Si/SiO₂ interface → RTS?
- Same statistics for simu. than exp.
- Inter-defects distance to define a cluster?
- Better model for the cross sections (real time TDDFT)
- Temperature dependance of the cross sections
- Other impacts of clusters (carrier trapping, recombination...)
- Include these results into predictive softwares

Questions time

Italy



ajay@laas.fr

Thank you for your attention



questions

suggestions

France
ST

life.augmented



LAAS
CNRS



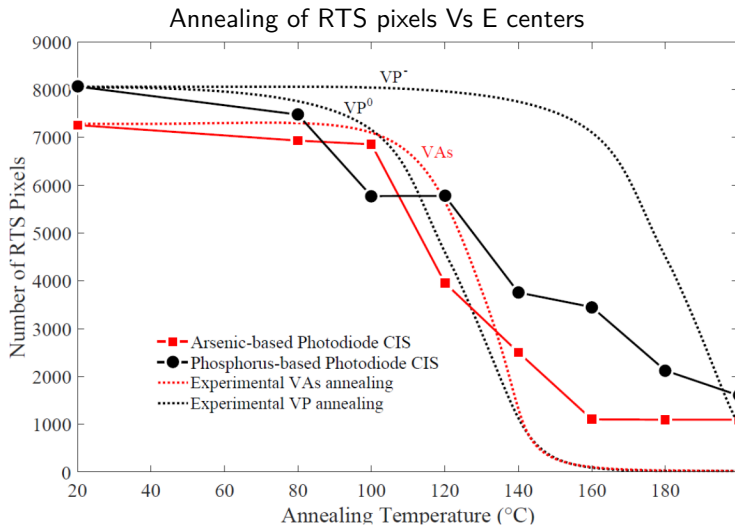
isae
Institut Supérieur de l'Aéronautique et de l'Espace
SUPAERO



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- 3 conclusion
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Effect of dopants?



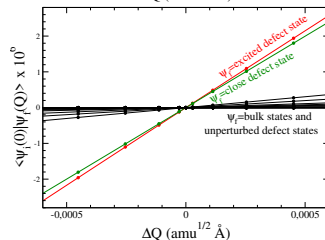
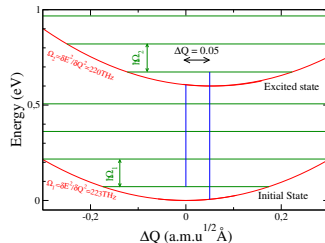
Dopants are not at the origin of DC-RTS.

Electronic cross section: scalar product

$$\begin{aligned}\langle\psi_i|\frac{\delta h}{\delta Q}|\psi_f\rangle &= (\epsilon_h - \epsilon_i) \langle\psi_i|\frac{\delta\psi_f}{\delta Q}\rangle \\ &= (\epsilon_h - \epsilon_i) \frac{\delta\langle\psi_i|\psi_f\rangle}{\delta Q}\end{aligned}$$

$$|\psi_f(\mathbf{R})\rangle = \sum_{k,nG} c_n e^{2i\pi(\mathbf{G}+\mathbf{k})\cdot\mathbf{R}}$$

$$\langle\psi_i|\psi_f\rangle = \sum_{k,nG} c_i c_f^*$$



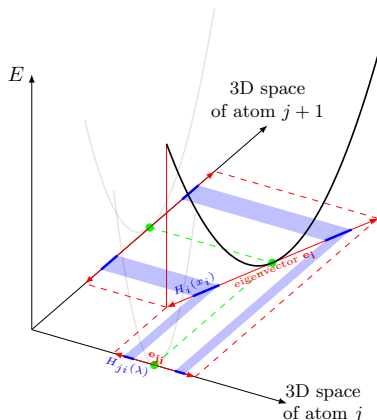
Representation of NDim ionic wf

- Harmonic approximation:
Solve Dynamical matrix and get eigenmodes \mathbf{e}_i
- Split ND H along each 1D \mathbf{e}_i :
 $\hat{H}^{ions} = \hat{H}_1 \otimes \hat{H}_2 \otimes \dots \otimes \hat{H}_{3N_{at}}$
- The exact solution is then
 $\chi(\mathbf{R}) = \prod_{i=1}^{3N_{at}} H_i(x_i)$

Approximation

Project the 1D solution on
the 3D space of each atom
memory = $N \times 100^3$ tabular

ionic wf do not overlap
Lose the relative motion
between atoms (not
needed)



Representation of NDim ionic wf

\forall atom j , \forall mode i :

$$\chi_0^j(\mathbf{R}) = \begin{cases} 1 & \text{if } \mathbf{R} = \mathbf{R}_{at_j} \\ 0 & \text{if } \mathbf{R} \neq \mathbf{R}_{at_j} \end{cases}$$

$$\chi_{i+1}^j(\mathbf{R}) = \int \chi_i(\mathbf{R} - \lambda \mathbf{e}_{ji}) H_{ji}(\lambda) d\lambda$$

$$\chi^j(\mathbf{R}) = \chi_{3N_{at}}^j(\mathbf{R})$$

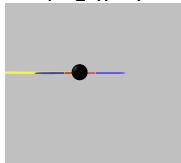
Example:

6 highest contribution modes.

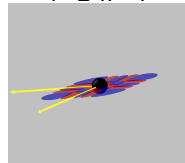
yellow= eigenmode

blue/red= +/- wf

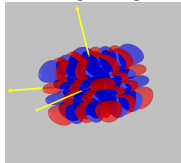
i=1 n=4



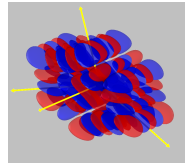
i=2 n=4



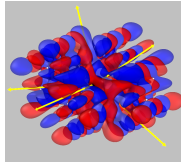
i=3 n=3



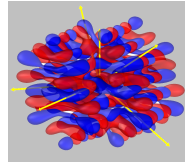
i=4 n=2



i=5 n=2



i=6 n=1



Ionic scalar product

no overlap between ions \rightarrow

The full ionic wf is the sum of each ionic projected contribution

$$\chi^{proj}(\mathbf{R}) = \sum_{j=1}^{N_{at}} \chi^j(\mathbf{R}) \quad (1)$$

The final scalar product is:

$$\langle \chi_{im} | \Delta \mathbf{Q}_{\mathbf{k}} | \chi_{fn} \rangle \sim \sum_{j=1}^{N_{at}} \langle \chi_i^j | \mathbf{Q}_{\mathbf{j}\mathbf{k}} | \chi_f^j \rangle \quad (2)$$